L = liquid phase
p = pressure
TP = two-phase flow
v = viscous
I = Phase I
II = Phase II
1 = for control volume 1
2 = for control volume 2

Greek Letters

Ψ = dynamical characteristic
μ = viscosity, $ML^{-1}T^{-1}$
ρ = density, ML^{-3}

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Stability of a Laminar Jet of Viscous Liquid—Influence of Nozzle Shape

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For a long time there has been an interest in the many diverse applications of laminar liquid jets and, consequently, their stability. The purpose of this study is to single out nozzle shape, since it has received little systematic attention in the past. In particular, it is shown that data for zero length orifices can be interpreted in terms of the better known properties of long cylindrical nozzles. A possible extension to nozzles of arbitrary shape and length is discussed. The fluids considered are Newtonian (constant viscosity coefficient), and the influence of the ambient atmosphere is included.

GENERAL DISCUSSION

For the initial portion of the breakup curve for cylindrical nozzles (the segment AB in Figure 1), breakup length is proportional to velocity. This segment has been much studied in the past and can be considered to be well understood. Experimental results are generally in agreement with each other and in turn are in agreement with theory. Grant and Middleman (1966) and Meister and Scheele (1967) give summaries of previous work and the progress to date. Almost all experimental data are concerned with long cylindrical nozzles.

The cause of the peak in the breakup curve (point C in Figure 1) has been a problem for some time. It seems likely that there is not a single cause but actually two competing ones. The nondimensional parameters that determine the onset of each of these causes are discussed below.

The relative motion of the jet and the ambient gas produces a pressure increment on the jet surface that tends to amplify small disturbances of the surface. The parameter that controls this process is the Weber number based upon the ambient gas density We_a. Fenn and Middleman (1969) identified this parameter and experimentally found the critical value to be $We_a = 5.3$. The theory of Weber (1931), that includes this ambient influence, can be simplified to the point that calculations can be made easily without making restrictive assumptions (Phinney, 1973). It is found that for low jet viscosity the critical Weber number is largely independent of viscosity and is close to the value given by Fenn and Middleman (1969). In addition, the theory predicts the shape of the breakup curve up to and beyond the peak. These predictions agree reasonably well with experimental observations.

The above discussion is based upon the assumption that the exit flow has a constant disturbance level at the jet exit, independent of the jet velocity. It is found, however, that for long cylindrical nozzles the disturbance level increases with velocity after some threshold value defined by a critical Reynolds number $\hat{Re}$ (Phinney, 1972). This increase in disturbance level seems to be connected with some Tollmien-Schicting type of instability that ultimately leads to transition and turbulent flow in the pipe.

For any particular jet from a long nozzle, the peak in the breakup length-velocity curve will be due to one of the above causes. Which cause is determined in a particular case by which of the critical velocities is the lowest.

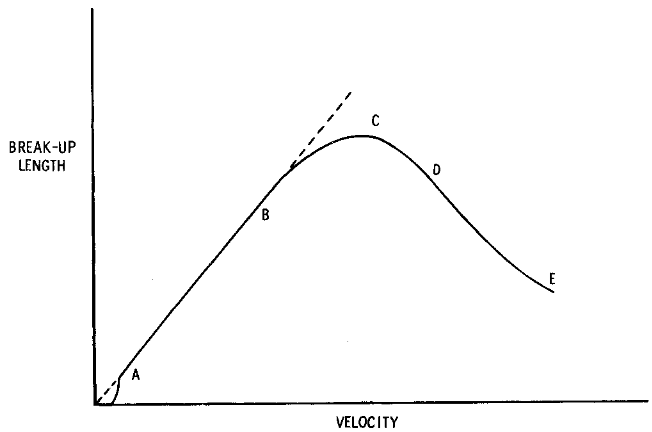

Fig. 1. Schematic of a breakup length-velocity curve.

For a jet from an orifice plate, the transition from laminar to turbulent flow occurs at much higher velocities than for a long nozzle. One would expect that this difference in behavior would be reflected in the exit disturbance as a function of the Reynolds number. It is found, in fact, that the critical Reynolds number $\hat{Re}$ for this orifice is so large that it is not generally observed. The controlling factor for the orifice data is normally the ambient Weber number We_a.

The above discussion is consistent with experimental results, for example, take the data shown in Figure 10 of Grant and Middleman (1966). The critical Reynolds number for the long nozzles is $\hat{Re} = 600$ which gives an exit velocity of 1130 cm/s. The critical $We_a = 5.3$ gives a velocity of 1530 cm/s. In other words, the short nozzle, which we presume to be sensitive to We_a only, departs from the linear relationship (L proportional to V) at a somewhat higher velocity than the longer nozzles. These values are seen to be consistent with Figure 10 and with Grant's statement "..., the short tube produces jets which are markedly more stable than those produced from longer tubes." How much more stable the short nozzles are will depend upon the magnitude of the difference in the two critical velocities.

EXPERIMENTAL DATA

Although there is much breakup length data for long nozzles, there is little information for flow from orifices. For the comparison made in the next section, orifice data were taken from two sources.

The first source is the thesis of Sterling (1969). He uses a conventional technique in which the jet breakup is measured photographically. Great pains were taken to ensure that sufficient data were collected (50 to 100 pictures at each condition) to obtain statistically valid averages. The apparatus was also constructed with unusual care to prevent ground vibration from being transmitted to the nozzle. The nozzle consisted of an ogive contoured section with an area contraction of more than 250 to 1. The ogive nozzle ended in a sharp exit with no appreciable straight section. A complete description of the apparatus and a tabulation of the data are found in Sterling's thesis.

The second source of data is that of Phinney and Humphries (1970). An electrical method is used for measuring breakup length. An average value for breakup length is obtained directly, permitting many data points to be obtained quickly. This is an unusual method, but when the results for long nozzles are compared with previous data, good agreement results.

The orifice itself was a hole cut in shim stock so that the length-to-diameter ratio was less than 0.1. The sudden area contraction in front of the orifice was at least 550 to 1. Although several cases were studied, only two gave sufficient data near and after the peak to warrant inclusion here. A complete description of the apparatus and a tabulation of the data are to be found in the report of Phinney and Humphries.

In the treatment of the data, no correction was made for the vena contracta. Generally speaking, this effect is not large enough (about a few percent) that it would change the nature of the conclusions. In addition, the changes in velocity and diameter of the jet are such that, in most parameters, they tend to some extent to compensate for each other. As more data become available for both the breakup length and the jet diameter change, a more careful consideration of this effect would be in order.

INTERPRETATION OF RESULTS

The mathematical theory makes no distinctions concerning the length or shape of the nozzle forming the jet. The parameters that it recognizes are the diameter after the vena contracta region and the disturbance level at this point (the effective exit conditions). Since cylindrical nozzles are observed to have a constant disturbance level over a considerable range of flow conditions, we are led to expect the same behavior for orifice plates. In fact, as explained below, since the orifice has a high critical Reynolds number, we might expect its disturbance level to be constant over a much broader range.

In order to eliminate for the moment the question of disturbance level changes for the cylindrical nozzles, consider only the data for which $Re < \hat{Re}$. The comparison between pipe and orifice is made in terms of $(L/D)/\sqrt{We_j}$, which is a nondimensional amplification rate and the parameter $\sqrt{We_a}$ which defines the magnitude of the destabilizing effect of the ambient gas. This choice is suggested by Weber's theory, as well as the success in correlating data; see Phinney (1973) for details. Data of Sterling (1969) and Phinney and Humphries (1970) are plotted in these coordinates and are shown in Figure 2. Only curves with We_a high enough to show the ambient influence have been presented in Figure 2. No restriction was made upon the Reynolds number. Points in Figure 2 have Re as high as 8,000.

The difference in horizontal asymptote for the data from the two different sources undoubtedly corresponds to the different initial disturbance levels achieved by the different experimenters. If the curves were adjusted vertically to compensate for this, it is seen that they are almost identical. It is also seen that the orifice data and the pipe data of Fenn and Middleman (1969) closely parallel each other, suggesting that the same mechanism is operating in each case. This mechanism is the destabilizing influence of the ambient atmosphere.

The two sets of orifice data have different upstream histories, one going through a sudden contraction and the other being accelerated smoothly. These differences seem to have little effect upon the stability of the jet as evidenced by the similarity of the data when plotted in Figure 2.

The close comparison of the orifice and pipe data brings into question Sterling's conclusion that the differences in stability are due to differences in the shape of their exit velocity profiles. The portion of the pipe data that does not correlate with the orifice is that for which the Reynolds number is above the critical $Re > \hat{Re}$ (the nozzle data that are not shown in Figure 2) where the disturbance level has started to increase. We see that it is

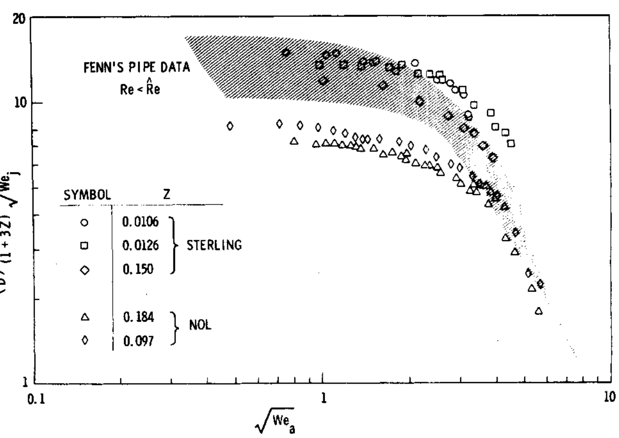

Fig. 2. Orifice data compared to constant disturbance level pipe data.

the disturbance level that is the primary characteristic which is different between orifices and longer nozzles. The vena contracta and velocity profile are different, but they are secondary effects.

Although a critical Reynolds number was not observed for the orifice data, there must be some upper limit to the constant disturbance level regime. The exit disturbance level for an orifice could be increased from its constant laminar level by a number of different processes. In an infinite reservoir the sink-like motion can depart from its ideal laminar form along the wall that contains the orifice. At sufficiently high velocity, a turbulent boundary layer is formed and swept through the orifice. The orifice velocity necessary to significantly increase the disturbance level of the flow is unknown, but it probably has a much higher critical Reynolds number than that of a pipe exit. Also if the orifice is supplied by a pipe, then that pipe can feed its turbulent flow into the orifice increasing its disturbance level.

On the basis of the above discussion, one is led to postulate that like long nozzles those of intermediate or zero length also have a critical Reynolds number for which the exit disturbance level will start to increase. The critical Reynolds number can depend upon the apparatus that supplies the nozzle, as well as the nozzle itself.

A determination of the critical Reynolds number must be made with the ambient pressure low enough to suppress that mode of breakup. There is some hope that, if sufficient data are obtained for different nozzles, the critical Reynolds number could be correlated as a function of length, as well as other geometric factors. Once the critical Reynolds number is known, then, presumably, the complete breakup characteristics of the jet can be predicted with reasonable accuracy.

CONCLUSIONS

When the exit velocity is low, the breakup characteristics of jets from an orifice closely parallel those from a long nozzle. One concludes that in this regime the exit disturbance level for both cases is approximately the same and that there is little if any influence of the jet formation process.

For high enough exit velocity, a peak in the breakup curve is found. For all orifices tested, the peak was due solely to the presence of the ambient gas; whereas, the long nozzle can be influenced by exit disturbance level increases as well. In those pipe nozzle cases ($Re > \hat{Re}$) for which the jet breakup is due to a disturbance level increase, the jets will appear to be less stable than the corresponding orifice. Otherwise, there is little difference between them; except for the fact that because of the frictional losses, the pipes require a larger driving pressure to achieve the same exit velocity.

For an orifice with very high exit velocity, there are several possible sources of turbulence upstream of the exit that could produce an increased disturbance level and a corresponding reduced breakup length. However, this does not seem to be a factor in any of the data shown in Figure 2.

One is led to conjecture that the breakup curve for short but finite length nozzles can be predicted once its critical Reynolds number $\hat{Re}$ is known. $\hat{Re}$ would depend upon nozzle shape and length-to-diameter ratio and to a lesser extent upon the Ohnesorge number Z. If this conjecture is true, the prediction of jet breakup would be considerably simplified. The correlation of data would be mainly concerned with the dependence of the critical Reynolds number $\hat{Re}$ upon nozzle geometry and fluid characteristics. The critical Reynolds number would need to be determined from tests in which the ambient density is low enough to be negligible. On the other hand, if the cases of interest have high ambient density, then the nozzle shape is of little importance since the breakup is not controlled by this factor.

It must be kept in mind that the theory as usually applied cannot give precise numerical predictions since the initial disturbance level is generally unknown beforehand. Nevertheless, guidelines can be fairly closely drawn since a value can be guessed which will probably give results for breakup length within a 50% error band. The critical velocity for the onset of the influence of the ambient atmosphere does not depend upon the disturbance level and as a result can be predicted with reasonable accuracy. It should be expected that for nozzles of arbitrary length or shape, a correlation would be even less precise due to the many factors involved. Typically, one might expect to establish certain trends or limits beyond which a factor no longer has any effect. Obviously, the general problem is not completely solved by the suggested approach, but it should be considerably simplified by concentrating attention on the essential features.

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NOTATION

D = jet exit diameter
L = mean breakup length
Re = Reynolds number, $VD\rho_j/\mu$
$\hat{Re}$ = critical Reynolds number for which the exit disturbance level starts to increase
V = jet exit velocity
We_a = ambient Weber number, $V^2D\rho_a/\sigma$
We_j = jet Weber number, $V^2D\rho_j/\sigma$
Z = Ohnesorge number, $\mu/\sqrt{D\sigma\rho_j}$
μ = viscosity coefficient of jet fluid
ρ_a = density of ambient gas
ρ_j = density of jet fluid
σ = surface tension

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